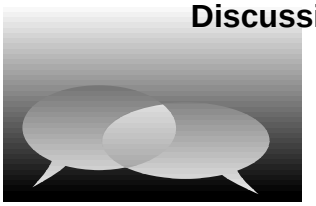


Discussion questions

A. Single-celled animals must passively absorb nutrients and oxygen from their surroundings, unlike humans who have lungs to pump air in and out and a heart to distribute the oxygenated blood throughout their bodies. Even the cells composing the bodies of multicellular animals must absorb oxygen from a nearby capillary through their surfaces. Based on these facts, explain why cells are always microscopic in size.

B. The reasoning of the previous question would seem to be contradicted by the fact that human nerve cells in the spinal cord can be as much as a meter long, although their widths are still very small. Why is this possible?

1.4 Order-of-Magnitude Estimates

> It is the mark of an instructed mind to rest satisfied with the degree of precision that the nature of the subject permits and not to seek an exactness where only an approximation of the truth is possible.
>
> Aristotle

It is a common misconception that science must be exact. For instance, in the Star Trek TV series, it would often happen that Captain Kirk would ask Mr. Spock, "Spock, we're in a pretty bad situation. What do you think are our chances of getting out of here?" The scientific Mr. Spock would answer with something like, "Captain, I estimate the odds as 237.345 to one." In reality, he could not have estimated the odds with six significant figures of accuracy, but nevertheless one of the hallmarks of a person with a good education in science is the ability to make estimates that are likely to be at least somewhere in the right ballpark. In many such situations, it is often only necessary to get an answer that is off by no more than a factor of ten in either direction. Since things that differ by a factor of ten are said to differ by one order of magnitude, such an estimate is called an order-of-magnitude estimate. The tilde, ~, is used to indicate that things are only of the same order of magnitude, but not exactly equal, as in

odds of survival ~ 100 to one .

The tilde can also be used in front of an individual number to emphasize that the number is only of the right order of magnitude.

Although making order-of-magnitude estimates seems simple and natural to experienced scientists, it's a mode of reasoning that is completely unfamiliar to most college students. Some of the typical mental steps can be illustrated in the following example.

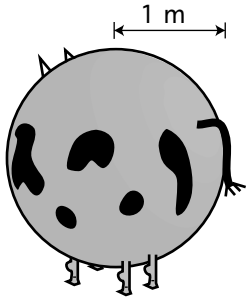
Example: Cost of transporting tomatoes
Question: Roughly what percentage of the price of a tomato comes from the cost of transporting it in a truck?

The following incorrect solution illustrates one of the main ways you can go wrong in order-of-magnitude estimates.

> **Incorrect solution**: Let's say the trucker needs to make a $400 profit on the trip. Taking into account her benefits, the cost of gas, and maintenance and payments on the truck, let's say the total cost is more like $2000. I'd guess about 5000 tomatoes would fit in the back of the truck, so the extra cost per tomato is 40 cents. That means the cost of transporting one tomato is comparable to the cost of the tomato itself. Transportation really adds a lot to the cost of produce, I guess.

The problem is that the human brain is not very good at estimating area or volume, so it turns out the estimate of 5000 tomatoes fitting in the truck is way off. That's why people have a hard time at those contests where you are supposed to estimate the number of jellybeans in a big jar. Another example is that most people think their families use about 10 gallons of water per day, but in reality the average is about 300 gallons per day. When estimating area or volume, you are much better off estimating linear dimensions, and computing volume from the linear dimensions. Here's a better solution:

> **Better solution**: As in the previous solution, say the cost of the trip is $2000. The dimensions of the bin are probably 4 m x 2 m x 1 m, for a volume of 8 m^3. Since the whole thing is just an order-of-magnitude estimate, let's round that off to the nearest power of ten, 10 m^3. The shape of a tomato is complicated, and I don't know any formula for the volume of a tomato shape, but since this is just an estimate, let's pretend that a tomato is a cube, 0.05 m x 0.05 m x 0.05, for a volume of 1.25×10^{-4} m^3. Since this is just a rough estimate, let's round that to 10^{-4} m^3. We can find the total number of tomatoes by dividing the volume of the bin by the volume of one tomato: 10 m^3 / 10^{-4} m^3 = 10^5 tomatoes. The transportation cost per tomato is \$2000/10^5 tomatoes=\$0.02/tomato. That means that transportation really doesn't contribute very much to the cost of a tomato.



Approximating the shape of a tomato as a cube is an example of another general strategy for making order-of-magnitude estimates. A similar situation would occur if you were trying to estimate how many m^2 of leather could be produced from a herd of ten thousand cattle. There is no point in trying to take into account the shape of the cows' bodies. A reasonable plan of attack might be to consider a spherical cow. Probably a cow has roughly the same surface area as a sphere with a radius of about 1 m, which would be $4\pi(1\text{ m})^2$. Using the well-known facts that pi equals three, and four times three equals about ten, we can guess that a cow has a surface area of about 10 m^2, so the herd as a whole might yield 10^5 m^2 of leather.

The following list summarizes the strategies for getting a good order-of-magnitude estimate.

(1) Don't even attempt more than one significant figure of precision.
(2) Don't guess area or volume directly. Guess linear dimensions and get area or volume from them.
(3) When dealing with areas or volumes of objects with complex shapes, idealize them as if they were some simpler shape, a cube or a sphere, for example.
(4) Check your final answer to see if it is reasonable. If you estimate that a herd of ten thousand cattle would yield 0.01 m^2 of leather, then you have probably made a mistake with conversion factors somewhere.

Summary

Notation

$\propto$ ..... is proportional to

$\sim$ ..... on the order of, is on the order of

Summary

Nature behaves differently on large and small scales. Galileo showed that this results fundamentally from the way area and volume scale. Area scales as the second power of length, $A \propto L^2$, while volume scales as length to the third power, $V \propto L^3$.

An order of magnitude estimate is one in which we do not attempt or expect an exact answer. The main reason why the uninitiated have trouble with order-of-magnitude estimates is that the human brain does not intuitively make accurate estimates of area and volume. Estimates of area and volume should be approached by first estimating linear dimensions, which one's brain has a feel for.

Homework Problems

1 ✓. How many cubic inches are there in a cubic foot? The answer is not 12.

2. Assume a dog's brain is twice is great in diameter as a cat's, but each animal's brain cells are the same size and their brains are the same shape. In addition to being a far better companion and much nicer to come home to, how many times more brain cells does a dog have than a cat? The answer is not 2.

3 ✓. The population density of Los Angeles is about 4000 people/km^2. That of San Francisco is about 6000 people/km^2. How many times farther away is the average person's nearest neighbor in LA than in San Francisco? The answer is not 1.5.

4. A hunting dog's nose has about 10 square inches of active surface. How is this possible, since the dog's nose is only about 1 in x 1 in x 1 in = 1 in^3? After all, 10 is greater than 1, so how can it fit?

5. Estimate the number of blades of grass on a football field.

6. In a computer memory chip, each bit of information (a 0 or a 1) is stored in a single tiny circuit etched onto the surface of a silicon chip. A typical chip stores 64 Mb (megabytes) of data, where a byte is 8 bits. Estimate (a) the area of each circuit, and (b) its linear size.

7. Suppose someone built a gigantic apartment building, measuring 10 km x 10 km at the base. Estimate how tall the building would have to be to have space in it for the entire world's population to live.

8. A hamburger chain advertises that it has sold 10 billion Bongo Burgers. Estimate the total mass of hay required to feed the cows used to make the burgers.

9. Estimate the volume of a human body, in cm^3.

10 S. How many cm^2 is 1 mm^2?

11 S. Compare the light-gathering powers of a 3-cm-diameter telescope and a 30-cm telescope.

S A solution is given in the back of the book. ★ A difficult problem.

✓ A computerized answer check is available. ∫ A problem that requires calculus.

12. S. One step on the Richter scale corresponds to a factor of 100 in terms of the energy absorbed by something on the surface of the Earth, e.g. a house. For instance, a 9.3-magnitude quake would release 100 times more energy than an 8.3. The energy spreads out from the epicenter as a wave, and for the sake of this problem we'll assume we're dealing with seismic waves that spread out in three dimensions, so that we can visualize them as hemispheres spreading out under the surface of the earth. If a certain 7.6-magnitude earthquake and a certain 5.6-magnitude earthquake produce the same amount of vibration where I live, compare the distances from my house to the two epicenters.